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Further thoughts on convective heat transport in a variable-viscosity fluid

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In a previous paper (Booker 1976) we found experimentally that the convective heat transport in a fluid with temperature-dependent viscosity decreased significantly as the ratio of the viscosities at the top and bottom boundaries increased. In this note, we show that this decrease in heat transport can be entirely accounted for by an increase in the critical Rayleigh number with variable viscosity.

In a recent paper (Booker 1976), we reported measurements of the heat transport by natural thermal convection in a fluid with strongly temperature-dependent viscosity confined between rigid horizontal plates. We found that, at a fixed value of the Rayleigh number based on the viscosity at the mean of the boundary temperatures, an increase of the ratio of the viscosities at the top and bottom boundaries resulted in a slight but significant decrease in the heat transport relative to a layer with constant viscosity. At a viscosity ratio of 300 this decrease was 12 %. In this note we shall show that this decrease can be entirely predicted from the effect of the variable viscosity on the critical Rayleigh number.

Table 1 summarizes our experimental data. The critical Rayleigh number R_c for each run is also listed. R_c is the Rayleigh number for the onset of convection in a layer with a linear vertical temperature profile such that the viscosities at the top and bottom boundaries are the same as in the convecting layer. In principle, R_c could be measured by keeping the temperature drop across the fluid layer fixed and varying the depth. In practice, however, such a measurement would be extremely difficult and the values in table 1 are calculated in a manner similar to that of Liang (1969).

With the usual non-dimensionalization, the governing equation for marginal stability of a temperature perturbation of horizontal wavenumber a is

$$-Ra^2\theta = f_{zz}(D^4-a^4)\,\theta+2f_z(D^2-a^2)^2\,D\theta+f(D^2-a^2)^3\,\theta, \qquad (1)$$

where D and the subscript z mean differentiation with respect to the vertical coordinate z and $f = \nu/\nu_0$ is the ratio of the kinematic viscosity ν to its value ν_0 at the mean of the boundary temperatures. The Rayleigh number is

$$R = \alpha g \Delta T d^3/\kappa\nu_0,$$

where α and κ are the thermal expansion coefficient and thermal diffisivity of the fluid (assumed temperature independent), g is the acceleration due to gravity, ΔT is the temperature difference across the layer and d is the layer depth. To first order, only the dependence of viscosity on the undisturbed linear temperature profile enters (1) and f is therefore a known function of z.

Run	$\nu_{\max}/\nu_{\min}$	R_c	R	N	N/N_0	N/N_T
1	2·47	1755	14900	2·72	0·993	1·000
2	2·70	1765	58700	3·99	0·991	1·000
3	1·41	1713	115000	4·85	0·997	0·998
4	2·84	1772	160000	5·34	1·001	1·011
5	4·11	1799	397000	6·87	0·997	1·012
6	9·70	1975	21000	2·92	0·967	1·007
7	10·4	1987	66500	4·03	0·967	1·009
8	9·75	1976	176000	5·31	0·969	1·009
9	31·4	2244	54700	3·72	0·943	1·018
10	114	2574	15200	2·56	0·930	1·043
11	127	2613	56100	3·55	0·893	1·006
12	113	2572	122000	4·47	0·904	1·014
13	152	2677	69000	3·72	0·883	0·996
14	152	2677	173000	4·86	0·891	1·011
15	262	2813	22300	2·77	0·903	1·039
16	300	2863	85400	3·87	0·865	1·000
17	284	2846	203000	5·01	0·878	1·014
18	304	2863	207000	5·04	0·879	1·016
19	287	2856	220000	5·11	0·876	1·012

TABLE 1. The Rayleigh numbers R, viscosity ratios $\nu_{\max}/\nu_{\min}$ and measured Nusselt numbers N for thermal convection in polybutene no. 8 are from Booker (1976). The critical Rayleigh numbers R_c for each run were calculated in the manner described in the text. N_0 and N_T are predicted by relations (2) and (4) respectively.

The sixth-order equation (1) is easily transformed into six first-order differential equations which can be numerically integrated by standard techniques. For a fixed wavenumber a, the boundary conditions (tangential and normal velocity components and temperature perturbation zero on each boundary) can be satisfied only for certain values of the eigenvalue R. We then vary the wavenumber to find the absolute minimum R, which is called R_c. The calculated values of R_c in table 1 increase as the viscosity ratio $\nu_{\max}/\nu_{\min}$ increases.

Rossby's (1969) measurements of the Nusselt number (ratio of total heat transport to conductive heat transport) for a fluid of high Prandtl number ν/κ with a small viscosity ratio are well represented for $R > 4000$ by the curve

$$N_0 = 0{\cdot}184R^{0{\cdot}281}. \tag{2}$$

Our results at a small viscosity ratio (runs 1–5) are in excellent agreement with this relation. In order to compare properly the variable-viscosity results with those for constant viscosity one needs to incorporate the fact that the critical Rayleigh numbers are different. One possibility is to assume that (2) is a special case of a more general relation

$$N = C(R/R_c)^{\beta}. \tag{3}$$

For constant viscosity, $R_c = 1708$ and (2) implies $C = 1{\cdot}49$. Booker (1976) found that β was independent of the viscosity ratio. We now show that C is also independent of the viscosity ratio.

Table 1 lists the ratio of the measured Nusselt number N to

$$N_T = 1{\cdot}49(R/R_c)^{0{\cdot}281}. \tag{4}$$

The mean value of N/N_T for the seventeen measurements excluding runs 10 and 15 is 1·008 with a standard deviation of 0·007. This should be compared with a drop of 12 % in N/N_0 at a viscosity ratio of 300. There is a clear tendency for high Rayleigh number experiments to have higher values of N/N_T, but no evidence of a trend with viscosity ratio. This could be a systematic effect associated with our correction for heat transport in the plastic sides of the apparatus. A simpler explanation is the lack of sufficient significant figures in relations (2) and (4). The probable errors in Rossby's data simply do not warrant more significant figures, however, and we conclude that N/N_T is not significantly different from unity. Thus for strong convection ($R \gtrsim 10R_c$) and viscosity ratios up to at least 300, (4) accurately predicts the heat transport and C is independent of the viscosity ratio. We conclude that the observed decrease in heat transport relative to a constant-viscosity fluid can be entirely attributed to the effect of the temperature-dependent viscosity on the critical Rayleigh number. This rather remarkable result means that, apart from the critical Rayleigh number, a strongly temperature-dependent viscosity has no effect on convective heat transport.

Experiments 10 and 15 have both a high viscosity ratio and a relatively low Rayleigh number. Booker (1976) suggested that their apparently anomalously high heat transport was a systematic error either in the measurements themselves or in the comparison with relation (2) in the low Rayleigh number range. We still have no adequate explanation for these two points, but it is clear that the effect of variable viscosity on R_c is not the answer. Both N/N_0 and N/N_T are 3 % higher than one would predict from the higher Rayleigh number points with similar viscosity ratios.

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